Impact of nonlinearity on the zero-mode lasing in optical lattices

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Introduction

- First experimental generation of a state bound to a vortex represented by a point defect in a 2D linear photonic graphene [1];

Zero-modes:

- Emerge after applaying distortion to the underlying lattice;
- Lie in the mid-gap at zero energy;
- Extremaly robust to the external perturbation;
- Managing the driving parameters can lead to an efficient lasing regime.

Motivation: We examine how the nonlinearity provides and affects lasing via zero-mode. Based on the results, we propose a new topological laser realisable in a multicore optical fibre [2].

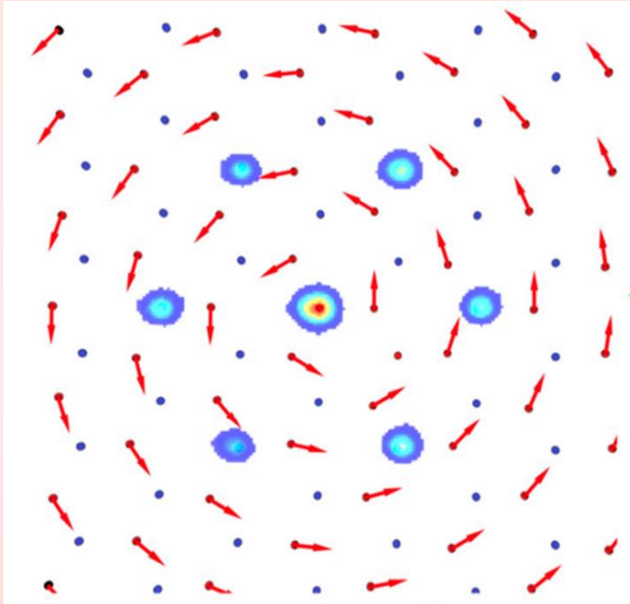

Fig. 1 Zero-mode indicated at the centre of the vortex presented in [1].

Model

- Hexagonal bipartite photonic lattice with armchair boundary conditions, described by the Hamiltonian $\widehat{H}$, which includes

$$\widehat{H} = \widehat{H}_{latt} + \widehat{H}_{Kerr} + \widehat{H}_{gain} \tag{1}$$

- Hopping between nearest neighbouring sites $\widehat{H}_{latt}$,

$$\widehat{H}_{latt} = -\sum_{<\vec{r},\vec{r}'>}\left(t + \delta t_{\vec{r},\vec{r}'}\right) + h.c. \tag{2}$$

- On-site saturable Kerr nonlinearity $\widehat{H}_{Kerr}$ [3],

$$\widehat{H}_{Kerr} = \frac{G}{2}\left[\frac{1}{1+|a_{\vec{r}}|^2}\hat{a}_{\vec{r}}^{\dagger}\hat{a}_{\vec{r}} + \frac{1}{1+|b_{\vec{r}}|^2}\hat{b}_{\vec{r}}^{\dagger}\hat{b}_{\vec{r}}\right] \tag{3}$$

- Non-Hermitian gain-loss effects $\widehat{H}_{gain}$,

$$\widehat{H}_{gain} = i\sum_{\vec{r}}\left[\frac{\Gamma_A}{1+|a_{\vec{r}}|^2}\hat{a}_{\vec{r}}^{\dagger}\hat{a}_{\vec{r}} + \frac{\Gamma_B}{1+|b_{\vec{r}}|^2}\hat{b}_{\vec{r}}^{\dagger}\hat{b}_{\vec{r}} - \gamma_A\hat{a}_{\vec{r}}^{\dagger}\hat{a}_{\vec{r}} - \gamma_B\hat{b}_{\vec{r}}^{\dagger}\hat{b}_{\vec{r}}\right]. \tag{4}$$

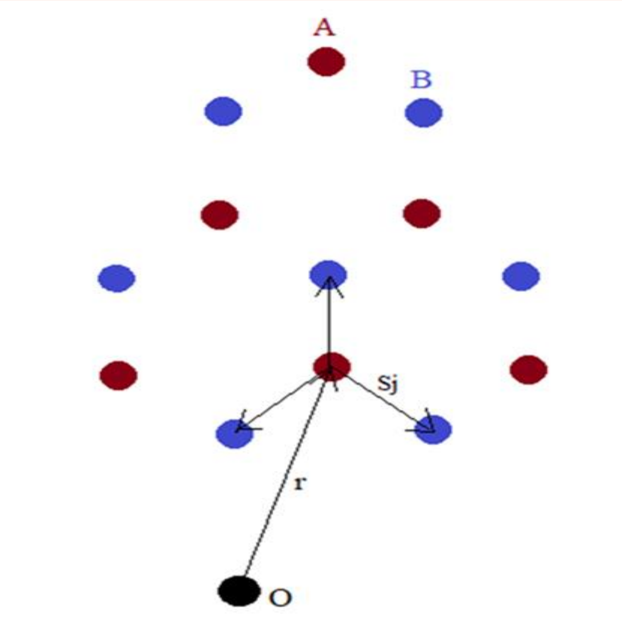

Fig. 2 Hexagonal lattice with bipartite symmetry.

Topologically protected zero-modes

- We consider a Kekule vortex-like distortion [4] to the coupling strength $\delta t_{\vec{r},\vec{r}'}$;
- It can be experimentally induced by small shifts in the waveguides' positions;
- The result of the distortion is the creation of a robust mode localized in the vicinity of the vortex core;
- The zero-modes consist of components distributed in each of sublattices, which are either localized around the vortex center (vortex component) or along the edge (edge component).

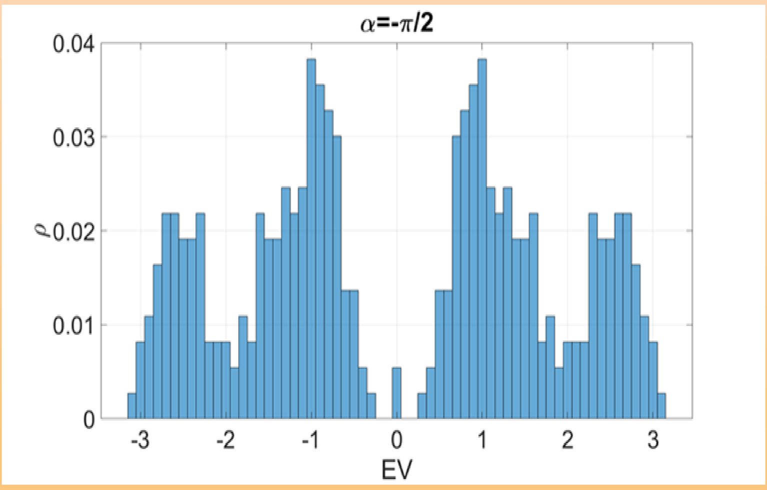

Fig. 3 Density of eigenstate distributions ρ.

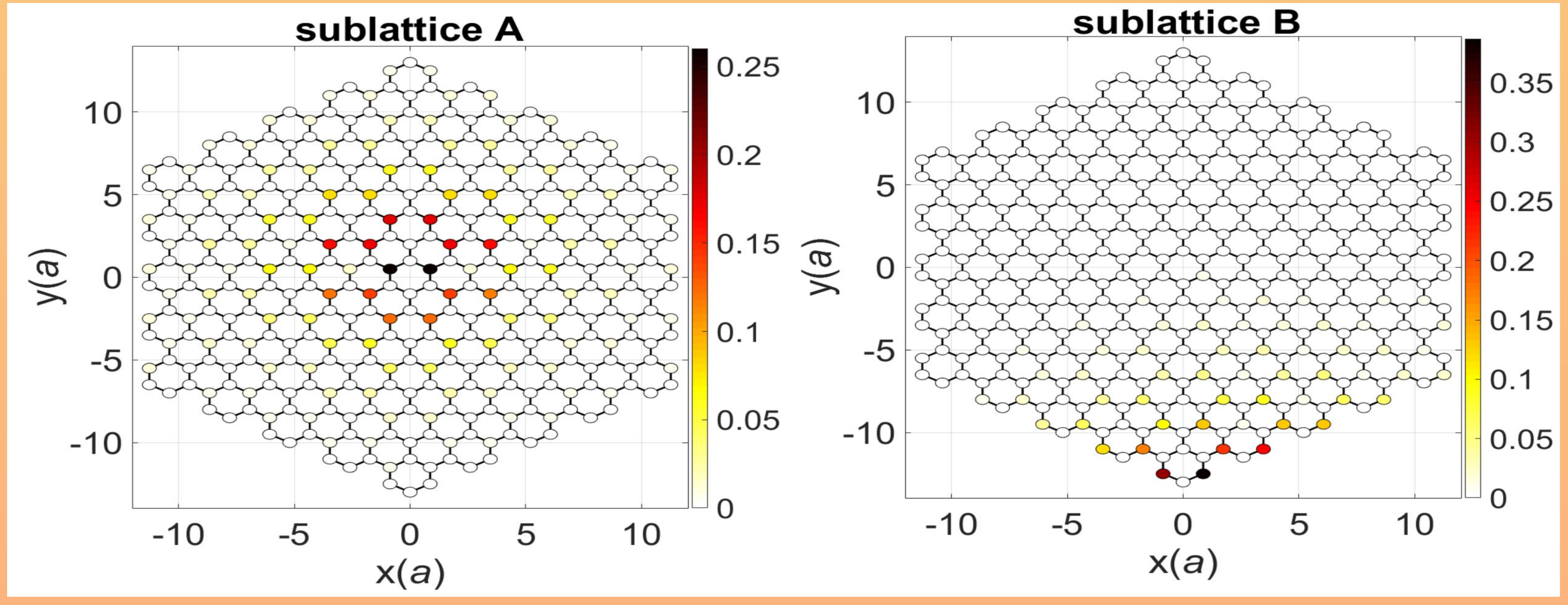

Fig. 4 Profiles intensity distributions of sum of all zero-modes over each of sublattices separately.

Nonlinear dynamics

- The zero-mode dynamics in the presence of local saturable Kerr nonlinearity $\widehat{H}_{Kerr}$, which induces breathing dynamics of the zero-modes by coupling of the central vortex and edge mode component;
- For sufficiently weak nonlinearity strengths |G| < 1, the fidelity oscillates in the range [0.9, 1], indicating weak nonlinearity-induced coupling between the vortex and edge zero-mode components and only weak mixing between the zero and bulk modes. For |G| > 1, the nonlinearity induced effects are significantly higher as indicated by reduction of the fidelity and destruction of the zero-mode.

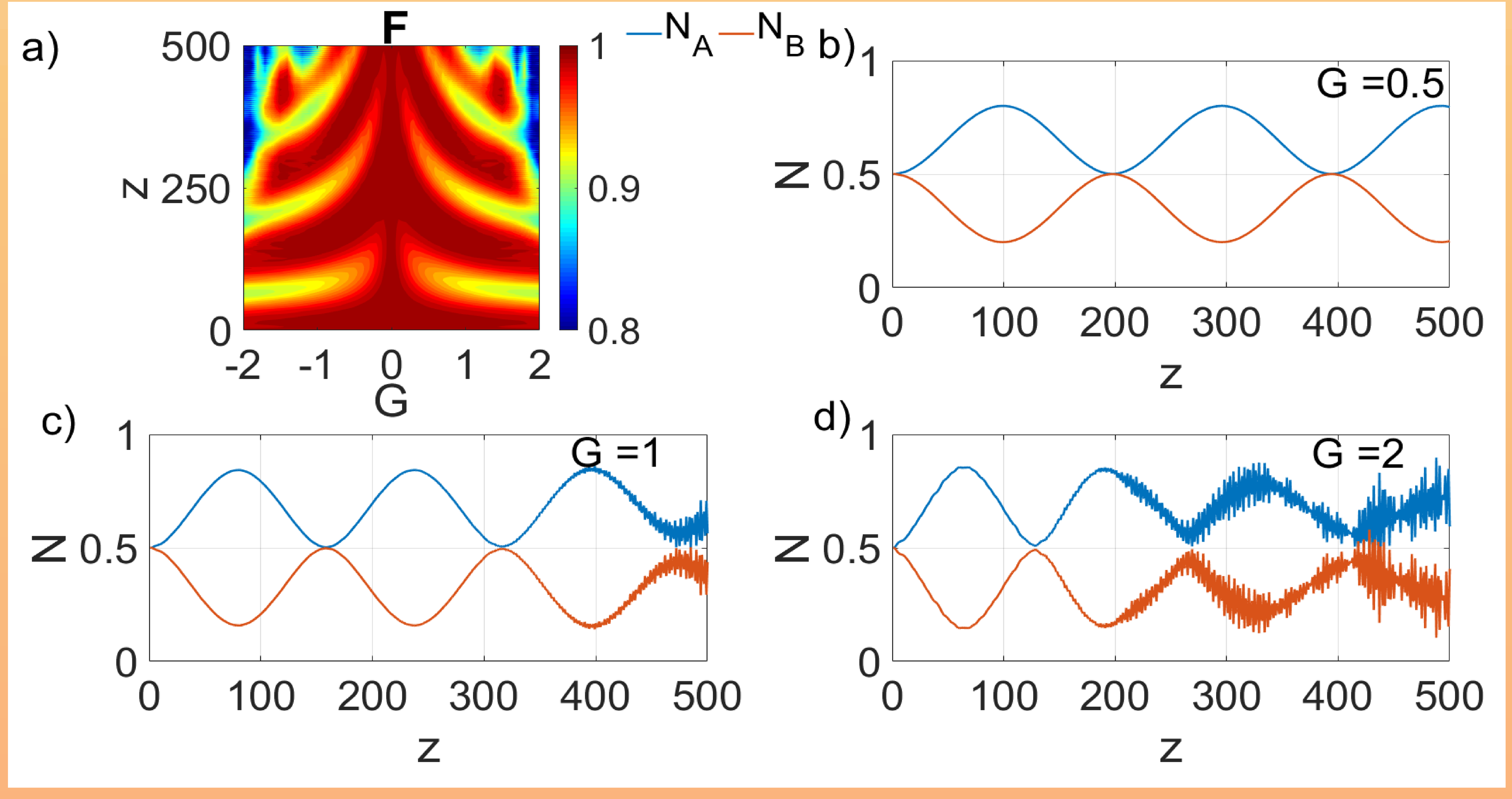

Fig. 5 The fidelity F between propagated and initial zero-mode through the nonlinear lattice vs. nonlinear strengths G. Plots in b) - d) show the norm evolution at certain values of G. Blue and red curves correspond to the mode components in sublattices A and B.

Zero-mode lasing

- Adding the saturable gain [5] and linear loss to lattice can lead to efficient lasing even from the noisy background;
- The sublattice A is pumped and sublattice B is lossy, corresponding to the parameters $\gamma_A = 0.01$, $\Gamma_A/\gamma_A = 10$, $\gamma_B/\gamma_A = 10$ and $\Gamma_B = 0$, the random initial state evolves into a zero-mode composed of the central vortex component on sublattice A, and a weaker component confiend to a corner of the sublattice B;
- Fidelity - the mode overlapping between normalized zero energy modes of $\widehat{H}_{latt}$, $\left|\psi_{\vec{r}}^{ZM}\right\rangle$ and normalized mode obtained during propagation $|\psi_{\vec{r}}(z)\rangle$.

$$F_{A/B}(z) = \sum_{\vec{r}}\left|\left\langle\psi_{\vec{r}}^{A/B}(z)\middle|\psi_{\vec{r}}^{(A/B)ZM}\right\rangle\right| \tag{5}$$

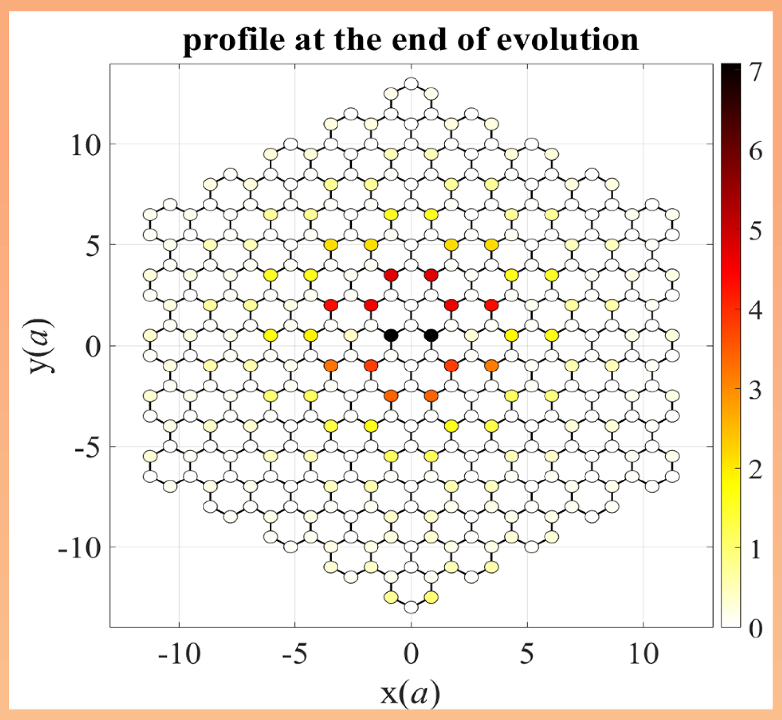

Fig. 6 An example of efficient lasing zero-mode.

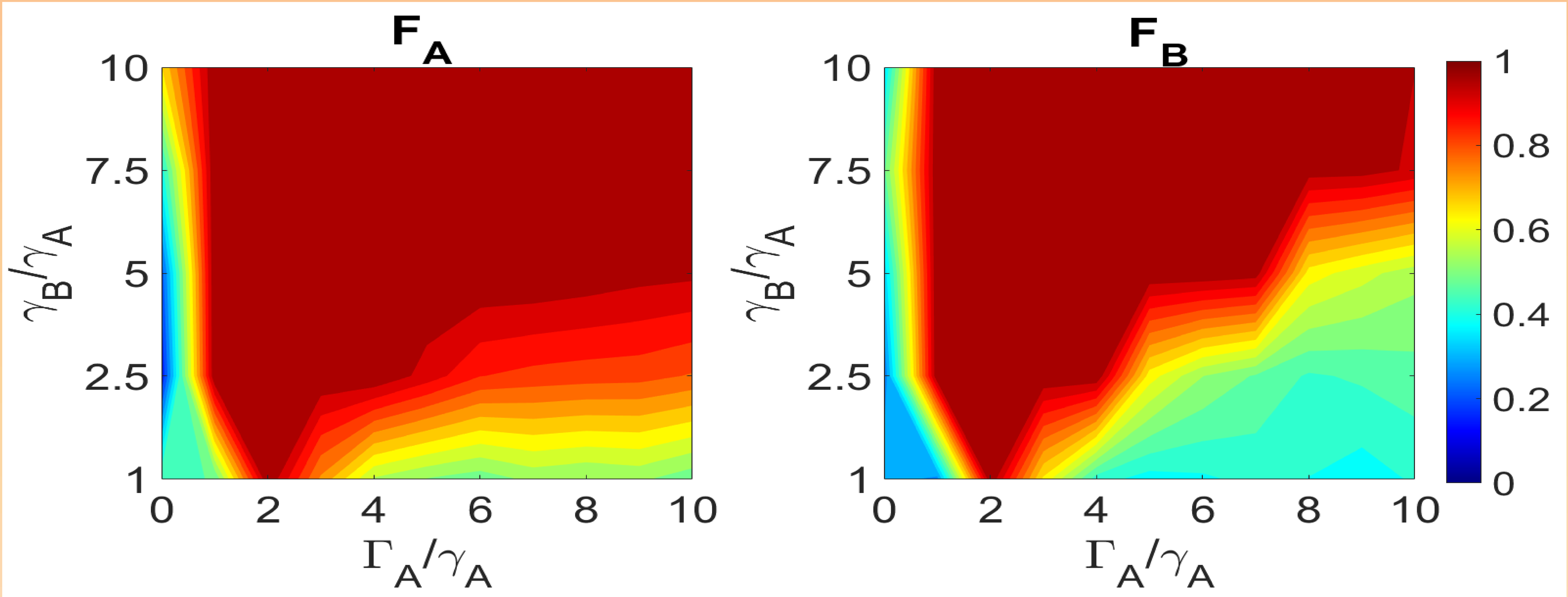

Fig. 7 The contour plot of fidelity at z = 1000 with respect to γ_B/γ_A and Γ_A/γ_A. Values of other parameters are fixed to $\Gamma_B = 0$, $\gamma_A = 0.01$.

Results

We can define three regimes for these concrete example and propagation length:

1) |G| < 0.1, robust vortex-edge-component lasing in the presence of small nonlinearity;
2) robust vortex-component lasing in the presence of medium nonlinearity 0.1 < |G| < 0.9;
3) delocalized radiation in the presence of high nonlinearity |G| > 0.9.

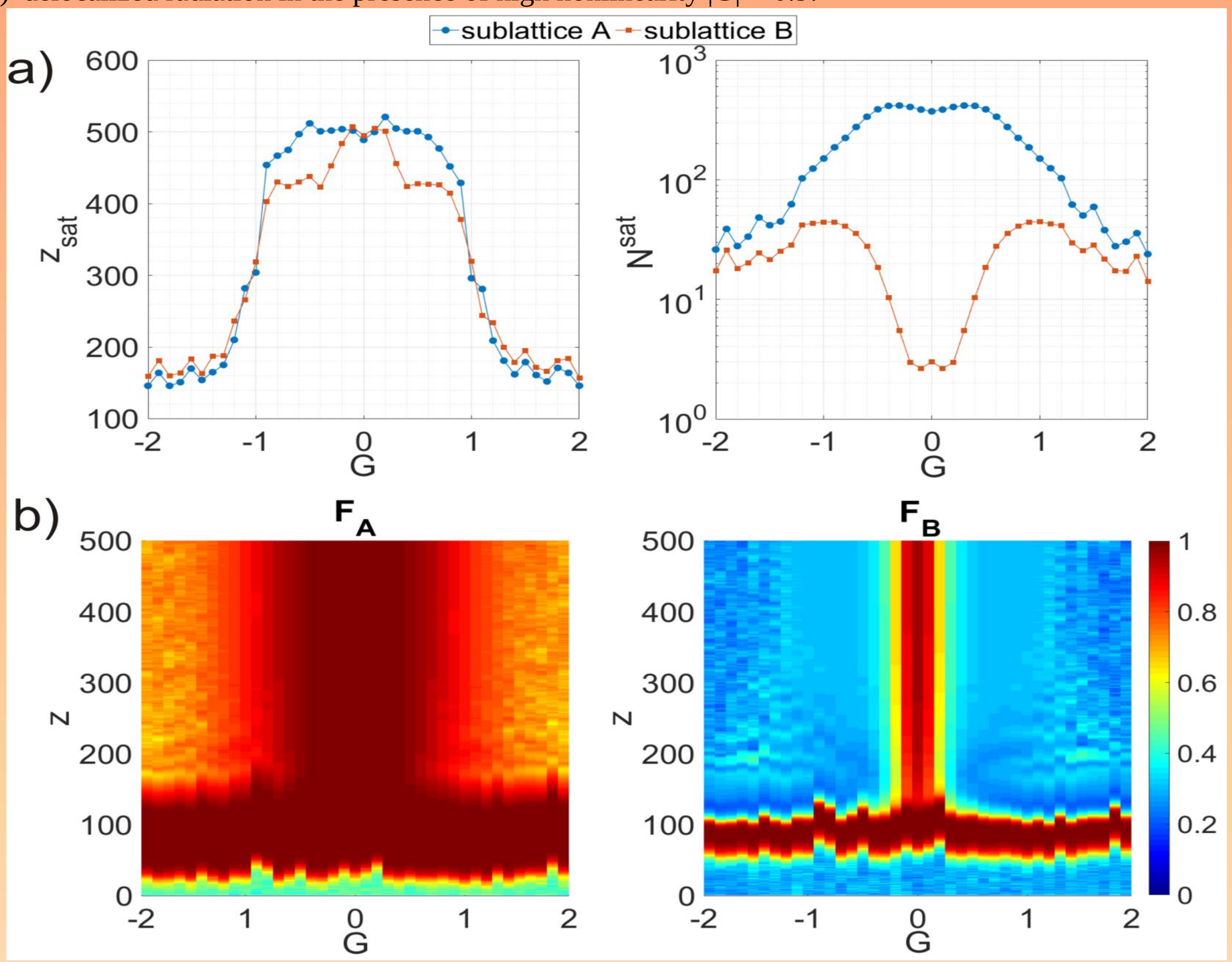

Fig. 8 The nonlinear hexagon-shaped lattice configuration with distortion: $\Gamma_A/\gamma_A = 10$, $\Gamma_B = 0$, $\gamma A = 0.01$, $\gamma B/\gamma A = 10$. a) z_{sat} and N^{sat} vs. G. b) Fidelity $F_{A/B}(z)$ during propagation vs. G.

Conclusion

- The nonlinearity-induced coupling between the vortex and edge mode components and the destabilization of the zero-mode in passive lattice occur also in the lasing regime in the active lattice;
- The results on high, destabilizing, nonlinearity indicate the advantage of large volume lasers based on vortex defects in which the finite dimensions of the lattice and thus coupling to the edge mode components, are less critical.

References

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