

# Supercell Metasurfaces: Hierarchical Designs and Experimental Validation

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## Introduction

Metasurfaces, consisting of an array of two-dimensional (2D) subwavelength nanostructures, provide unprecedented control over light at the nanoscale. By carefully engineering the size, shape, and arrangement of these nanostructures, metasurfaces offer precise manipulation of light properties. This capability addresses the longstanding challenge of achieving precise control over phase, amplitude, and polarization simultaneously [1].

Generally, metasurfaces are obtained through subwavelength unit cells designed to impart a local phase profile to light propagating through the cell. Using supercells (arrangements of multiple cells which are optimized together in the design), it is possible to achieve multiple functions at same time [2]. When the supercell is repeated periodically, the device behaves as a grating, splitting the light in multiple diffraction orders (*metagrating*). The size of the supercells determines the number and direction of diffraction orders to be considered while the geometry of the nanostructures present in the unit cell determines the local effect of the metasurface on light.

In this work, we demonstrate a new concept of metasurfaces based on a generalization of supercells. In general, these supercells consist of a series of unit cells that combine with each other to achieve a specific function. We present hierarchical supercells (supercells of supercells) and we extend supercells to general 2D Bravais Lattice, providing a simple mathematical framework to describe the resulting metagratings.

## Matrix Theory of Superlattices

Starting point: Bravais lattice defined by  $\mathbf{a}$  and  $\mathbf{b}$

$$\mathbf{r} = m\mathbf{a} + n\mathbf{b} = \begin{pmatrix} ma_x - nb_x \\ ma_y - nb_y \end{pmatrix} = \begin{pmatrix} a_x & b_x \\ a_y & b_y \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \hat{L} \begin{pmatrix} m \\ n \end{pmatrix}$$

Each and every point is identified by an integer pair  $(m, n)$ , and the lattice vectors are expressed by the lattice matrix  $\hat{L}$ . We can select a superlattice (which is actually a subset of the initial lattice) using an integer matrix  $S$  applied to the lattice points:

$$\begin{pmatrix} m \\ n \end{pmatrix} = \hat{S} \begin{pmatrix} m' \\ n' \end{pmatrix} \rightarrow \mathbf{r} = \hat{L}\hat{S} \begin{pmatrix} m' \\ n' \end{pmatrix}$$

The new lattice matrix is the product  $\hat{L}\hat{S}$ , and thanks to the associative property it can be applied to different hierarchical levels. Figure 1 shows for example:

$$\mathbf{r} = \hat{L}\hat{S}_1\hat{S}_2 \quad \hat{L} = \begin{pmatrix} 1 & 0.1 \\ 0.2 & 1 \end{pmatrix}$$

$$\hat{S}_1 = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \quad \hat{S}_2 = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

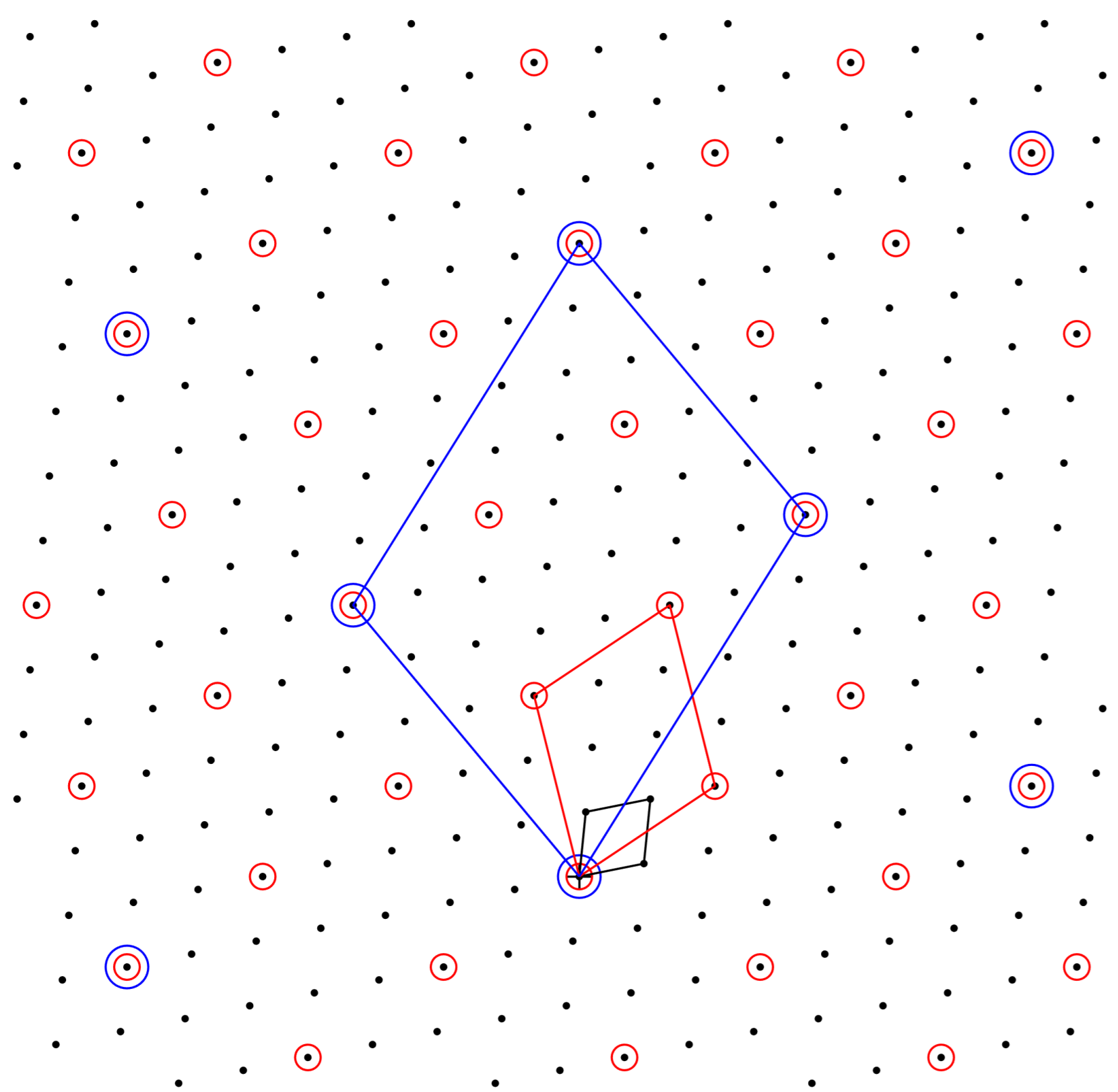


Figure 1. Example of hierarchical superlattices: The fundamental lattice is represented by black points while red and blue circles represent level 1 and 2 of the hierarchy, respectively.

Determinant of matrices  $S$  and  $L$  corresponds to:

- $|\det(\hat{L})|$  is the area of the unit cell
- $|\det(\hat{S})|$  is the number of unit cells in the supercell

## Metasurface Hierarchy

Using the aforementioned theoretical framework, we propose (Figure 2):

- Hierarchical supercell metasurface formed by supercells of supercells. Here we use  $\hat{S}_1 = 2$  and  $\hat{S}_2 = 2$
- Supercell based on a general integer Bravais lattice. Here  $\det(S) = 4$ , so there are 4 cells in the supercells

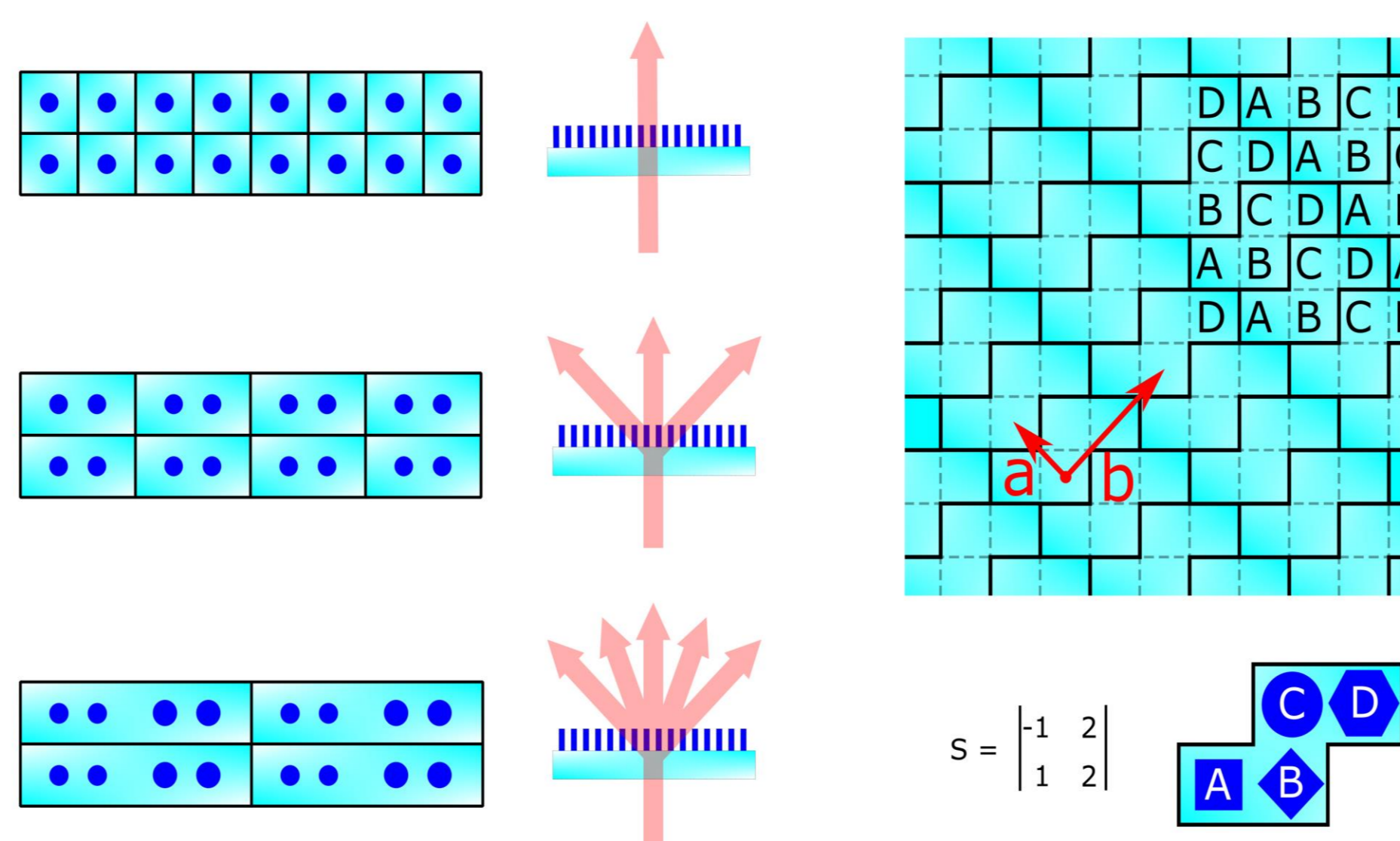


Figure 2. Generalized Supercell Metasurface concepts. LEFT: Metagratings using supercells of increasing hierarchy. 0<sup>th</sup>-order supercells are just the unit cells of the metasurface, 1<sup>st</sup>-order are created joining 0<sup>th</sup> cells and so on. Increasing hierarchies generate an increased number of diffraction orders. RIGHT: An example of supercell based on a general integer Bravais lattice

Each time the supercell order is increased, new diffraction orders are obtained. The phase and amplitude of each order depend on the particular asymmetry introduced when the supercell is created, specifically they are proportional to the perturbation parameters in the small-perturbation limit.

The diffraction orders for a metagrating with periodicity of  $P$  are given by:

$$\sum_{m,n} \delta \left( \bar{k} - \frac{2\pi}{P} \begin{bmatrix} m \\ n \end{bmatrix} \right)$$

while when the supercell is defined, the new diffraction orders (more numerous) are given by

$$\sum_{m,n} \delta \left( \bar{k} - \frac{2\pi}{P} L^{-1,H} \begin{bmatrix} m \\ n \end{bmatrix} \right)$$

## Simulations

We considered two simple initial designs

- A supercell formed by two cells, where asymmetry is introduced creating a lateral offset or a size offset in opposite directions in the pillars
- A metasurface having a supercell that implements a general Bravais lattice as described above

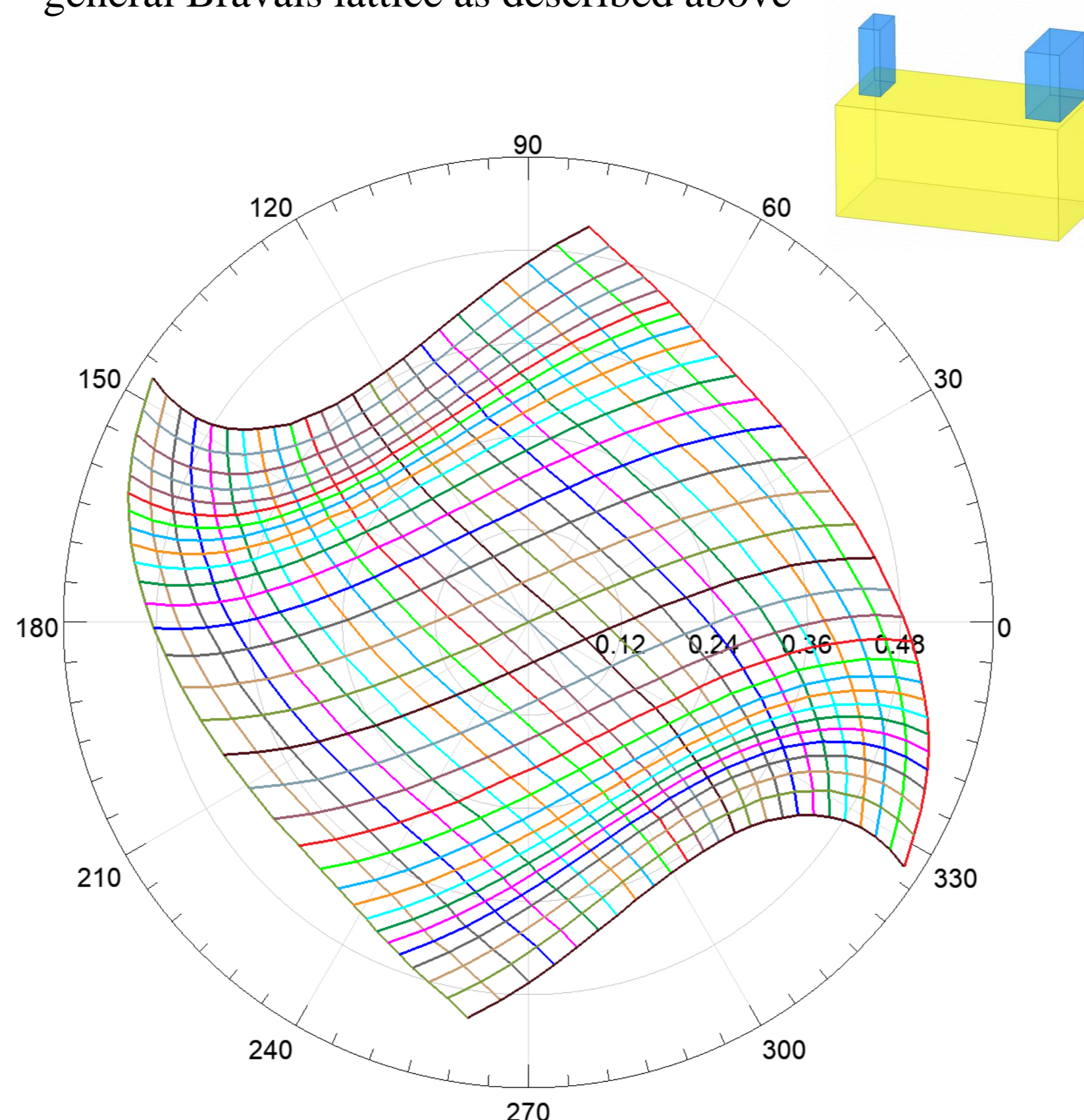


Figure 3. The complex amplitude of light on the diffraction order as a function of two perturbative parameters. For any given required phase and amplitude, it is possible to find a supercell to implement them.

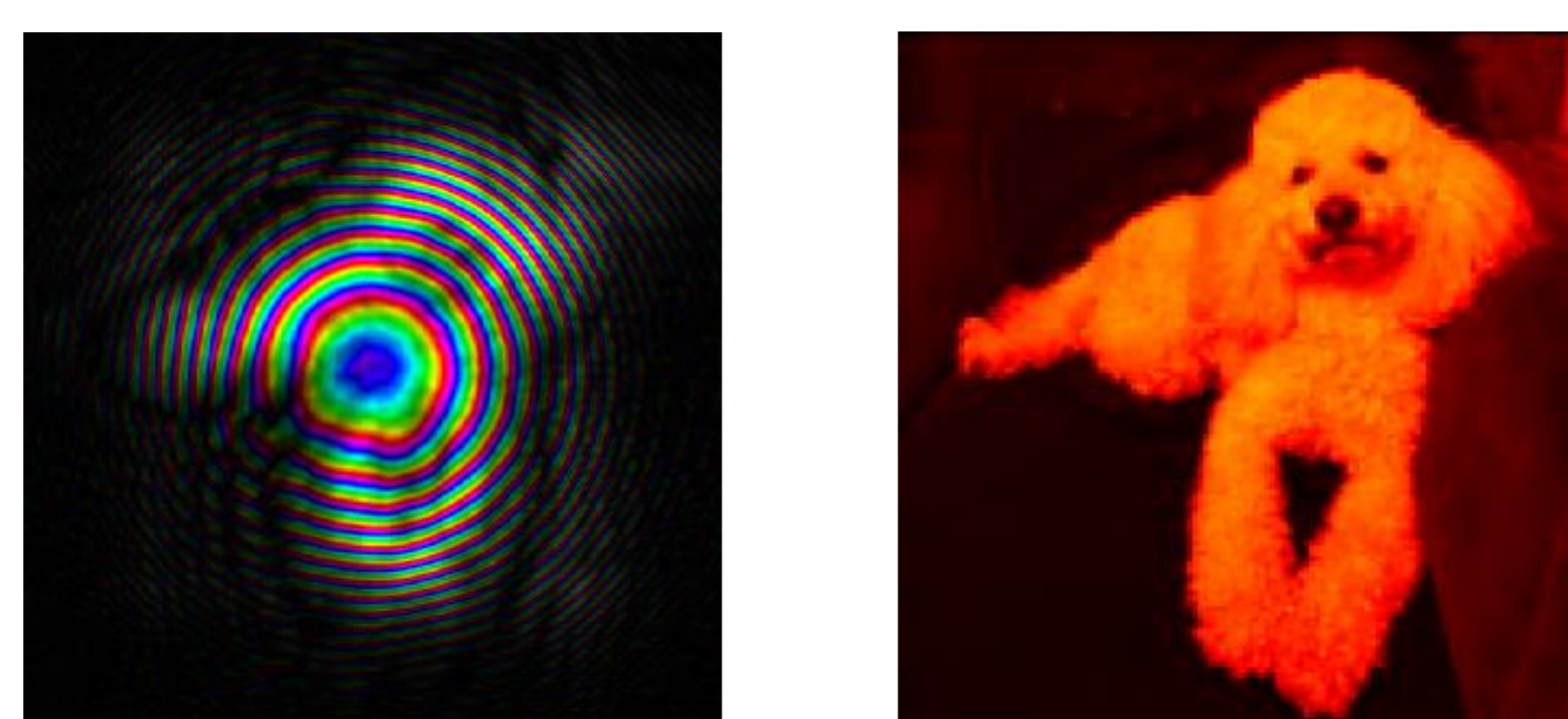


Figure 4. Phase and amplitude hologram and simulated image projected on a screen.

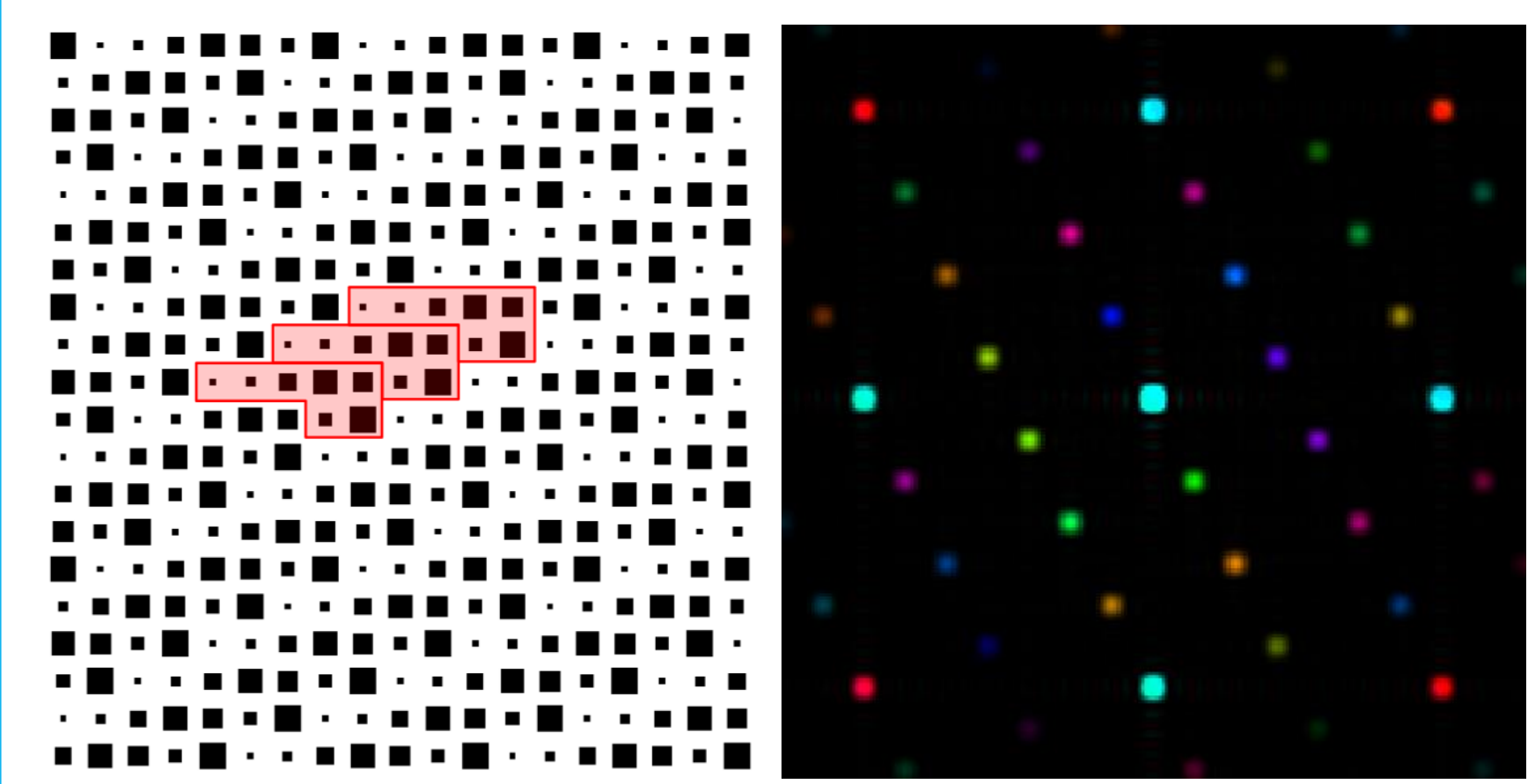
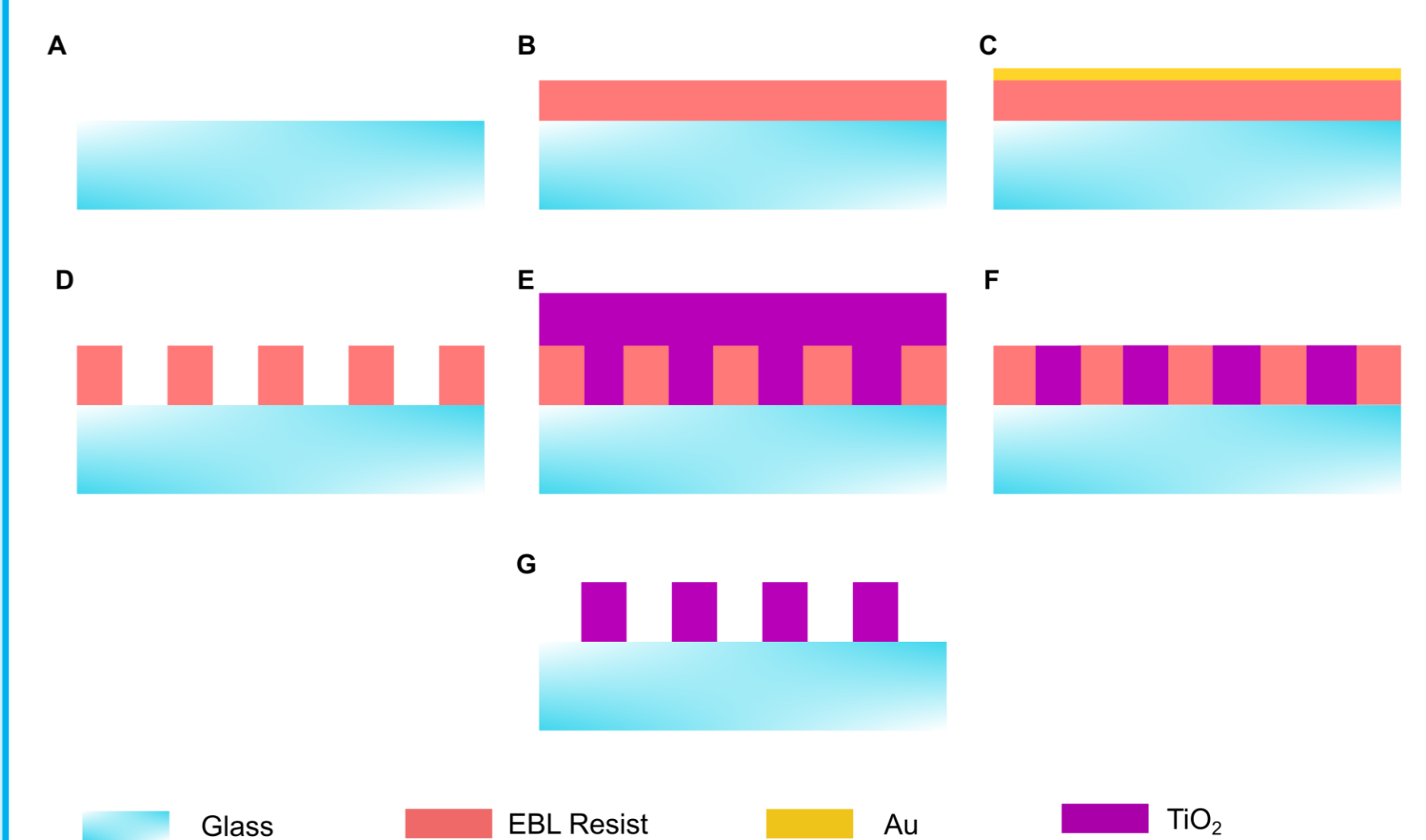


Figure 5. Example of diffraction pattern generated by a Bravais lattice supercell metasurface. The Teris-like supercell is highlighted in red. Different hues indicate different phases.

## Experimental Approach

We are currently fabricating the devices using the following process:



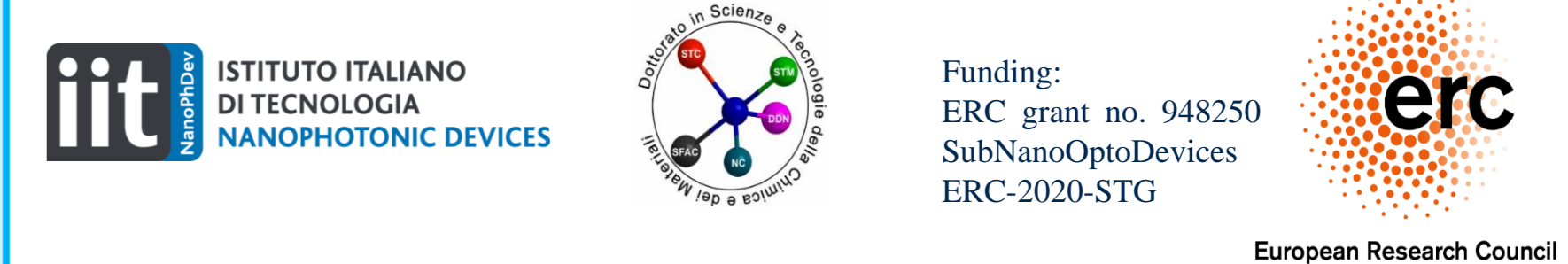
- Initially glass substrate is cleaned with solvents
- EBL resist spin coating and baking
- Sputter-coating with 10 nm of Au
- E-beam lithography, gold etching, and development
- Low-temperature ALD of  $\text{TiO}_2$
- Removal of excess oxide from ICP-RIE
- Resist strip using Remover PG

## Conclusions

- We demonstrated that hierarchical metasurfaces have many potentialities for beam shaping and holography.
- The presented techniques can be combined to create separate holograms on different orders.
- Independent control of phase and amplitude can be achieved to suppress speckle patterns.

## Acknowledgments

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## References

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